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A note on brane creation

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Abstract

The M-theory origin of brane creation processes is discussed.

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Recently Hanany and Witten[1] have shown how to derive non-perturbative results for (three-dimensional) field theory starting from ten-dimensional type IIB. In the course of the discussion they derived certain brane transition rules which show that when certain branes cross, other branes are created. This phenomenon has been the subject of some discussion [2],[3],[4],[5],[6]. In this paper we will discuss the M-theory origin of these transition rules. In particular the correct normalizations of various terms in the action of five and two branes coupled to M-theory obtained in [7], [8], will enable us to fix directly the relations between the charges and intersection numbers that enter into these rules.

We work in eleven dimensional Planck units, i.e. we have set $2\kappa_{11}^2 = 1$ so that the M-membrane tension is $T_2 = (2\pi)^{2/3}$ and the five-brane tension is $T_5 = (2\pi)^{1/3}$.¹ The action I for low-energy M theory coupled to a two-brane and a five-brane allowing also for the possibility of the two-brane ending on the five-brane is²,

$$\begin{aligned} \frac{I}{2\pi} = & -\frac{1}{2}(2\pi)^{-\frac{1}{3}} \int_M x_4 \wedge *x_4 + \frac{1}{3} \int_M c_3 \wedge x'_4 \wedge x'_4 - \int_M x_4 \wedge \left(\frac{1}{2}x'_4 \wedge c_3 - \Omega_7 + \theta_7\right) \\ & - \int_{W_3} c_3 + \int_{\partial W_3} b_2 - \frac{1}{4} \int_{W_6} h \wedge *h - \int_{W_6} \frac{1}{2} x'_4 \wedge b_2, \end{aligned} \quad (0.1)$$

In the above M is the eleven-manifold, W_3 is the (open) world volume of the two-brane whose boundary sits on W_6 (the world volume of the five-brane). Also³ $x_4 = x'_4 + \theta_4$, where the second term is a coexact four-form that solves $dx_4 = d\theta_4 = -\delta_5(M \rightarrow W_6)$. In the neighbourhood of the five-brane the M has the (twisted) product form $W_6 \otimes D_5$ where D_5 is an open five disc bounded by a four sphere and the delta function is normalized such that $\int_D \delta = \int_{S_4} x_4 = 1$. It should be noted that the fields in the above action are normalized such that $\int x'_4$ over any four cycle is integral. Also $h_3 = h'_3 - c_3 \dots$ where $h'_3 = db_2$ locally, and the normalization is such that $\int h'_3$ over any three cycle in W_6 is integral. The ellipses in the expression for h_3 come from the presence of sources on the

¹These results follow from the Dirac quantization condition and the relation $T_2^2/T_5 = 2\pi$ and are reviewed in [8].

²The complete form of this action as written here is given in [8]. It depends heavily on the work of other authors, particularly [7]. A complete (to the authors knowledge) list of references is given in [8].

³The terms Ω_7 and θ_7 are related to perturbative anomaly cancellation coming from five-brane anomalies. They are irrelevant for us and hence will be ignored in the rest of this paper.

five-brane due to the ends of two-branes. This is fixed by comparing with the equation of motion coming from the above action and imposing the self-duality constraint $*h = h$.

The important point about this action is that it couples both the two brane, and its magnetic dual the five-brane, to the M-theory background given in its normal form (i.e. with just the three form gauge field). It also contains the terms relevant to having 2-branes with boundaries on 5-branes. In this sense there is no analogue in string theory effective actions or four dimensional field theory of charges and monopoles. To derive the equations of motion it is convenient to rewrite the last four integrals as follows.

$$\begin{aligned}
& - \int_M c_3 \wedge \theta_8(M \rightarrow W_3) + \int_M b_2 \wedge \delta_5(M \rightarrow W_6) \wedge \delta_4(W_6 \rightarrow \partial W_3) \\
& - \frac{1}{4} \int_M (h \wedge *h) \wedge \delta_5(M \rightarrow W_6) + \int_M \frac{1}{2} x'_4 \wedge b_2 \wedge \delta_5(M \rightarrow W_6)
\end{aligned} \tag{0.2}$$

In the above the δ_r denotes delta function r-forms with the indicated support, and θ_r is an r-form which is a product of theta functions and delta functions which restrict M to the open manifold W_3 . This function satisfies $d\theta_8(M \rightarrow W_3) = -\delta_9(M \rightarrow \partial W_3)$ (see for example section 6 of [8]). The variation with respect to b_2 then gives,

$$d * h = dh = -x'_4|_{W_6} - 2\delta_4(W_6 \rightarrow \partial W_3) \tag{0.3}$$

where in the first equality we used the self duality of h . It is also instructive to derive the c_3 equation of motion.

$$(2\pi)^{-\frac{1}{3}} d * x_4 = -\frac{1}{2} x'_4 \wedge x'_4 - \theta_8(M \rightarrow W_3) - \theta_4 \wedge x'_4 + \frac{1}{2} h \wedge \delta_5 + \frac{1}{2} (db_2 - c_3) \wedge \delta_5 \tag{0.4}$$

It may be checked that this satisfies the consistency condition $d^2 * x_4 = 0$ once one uses the Bianchi identity for h (0.3) and the equation $d\theta_4 = -\delta_5$.

Let us now consider the case of several three-branes ending on a given five-brane. In this case the second term on the right hand side of (0.3) has to be replaced by a sum over all the string sources on the five-brane, coming from the boundaries of the membranes. Thus we get

$$dh = -x'_4|_{W_6} - 2 \sum_i e_i \delta_4(W_6 \rightarrow \partial W_3^i) \tag{0.5}$$

where $e_i = \pm 1$ are the (normalized) charges carried by these sources. Integrating over any four cycle Y_4 in W_6 we get,

$$\int_{Y_4 \subset W_6} x'_4 = 2 \sum_{i \subset Y_4} e_i \quad (0.6)$$

Note that the last sum is only over those end strings that are contained within Y_4 . Now x'_4 is the ambient field at W_6 which may have as sources all other five-branes apart from the one that is being considered. In particular the above integral is the intersection number of all five-branes threading the four cycle Y_4 . For instance if a five brane with world volume W_6^i threaded the four cycle then,

$$\int_{Y_4} x'_4 = \pm \int_{D_5} dx'_4 = \pm \int_{D_5} \delta(M \rightarrow W_6^i), \quad (0.7)$$

where $D_5 \subset M$ is an open disc with boundary $\partial D = Y_4$ except for orientation which may be the same or opposite. Thus the equation (0.6) can be written as,

$$Q = \frac{1}{2} \sum_{i \subset D_5} e_i^{(5)} - \sum_{i \subset Y_4} e_i^{(2)} = 0 \quad (0.8)$$

where $e_i^{(5)} = \pm 1$ are the five-brane charges and the first sum extends over all five branes intersecting D_5 .

The above considerations are valid for the compact case. In the non-compact case which is considered in the literature ([1]) the integral $\int dh$ is not zero but is an integral over the surface at infinity and so Q is a constant rather than zero, as in the analogous case considered in [1]. The important point about the formula (0.8) is that the correct relative normalizations of the different terms which is a consequence of gauge invariance and the tension formulae, guarantee that the five-brane intersections contribute half as much as the end strings which bound the membranes on the five-brane.

The M-theory relation (0.8) implies similar relations for various string theory configurations by various S- and T-duality transformations. For instance let us consider the M theory configuration of two five-branes with a two-brane suspended between them (i.e. with each of two boundaries sitting on one of the five-branes). The coordinates in parentheses are the spatial directions in which the branes are extended. (The M-coordinates

are $(x^0, x^1, \dots, x^{10})$.)

$$M : \quad 5M (x^1, x^3, x^4, x^5, x^{10}); \quad 2M (x^1, x^6), \quad 5M (x^1, x^2, x^7, x^8, x^9) \quad (0.9)$$

By wrapping x^{10} around a circle and letting its radius shrink to zero we have the corresponding IIA configuration of a two(D)-brane suspended between a four(D)-brane and a five(NS)-brane.

$$IIA : \quad 4D (x^1, x^3, x^4, x^5); \quad 2D(x^1, x^6); \quad 5NS (x^1, x^2, x^7, x^8, x^9) \quad (0.10)$$

By T-dualizing along x^2 we then get the following IIB configuration.

$$IIB : \quad 5D (x^1, x^2, x^3, x^4, x^5); \quad 2D(x^1, x^2, x^6); \quad 5NS (x^1, x^2, x^7, x^8, x^9) \quad (0.11)$$

This is the (S-dual of the) configuration considered by Hanany and Witten. Successive T- and S- dualities (U-duality) will enable us to generate many different configurations considered in the literature. For instance as pointed out in [5] starting from the last configuration, one can first T-dualize with respect to (x^1, x^2) then S dualize and then T-dualize with respect to (x^3, x^4, x^5) to get the IIA configuration of a 0D-brane and a 8D-brane with a fundamental string suspended between them. This can also be obtained directly from the following M-theory configuration.

$$M : \quad 5M (x^1, x^2, x^3, x^4, x^{10}); \quad 2M (x^5, x^{10}) \quad 5M (x^6, x^7, x^8, x^9, x^{10}) \quad (0.12)$$

By compactifying x^{10} on S^1 we get two 4D-branes with fundamental string suspended between them, and then T-dualizing with respect to (x^6, x^7, x^8, x^9) we get an 8D-brane and a 0D-brane with an F-string suspended between them. This result however seems to indicate that the latter configuration which is contained in massive IIA can be obtained from M-theory which does not permit a cosmological constant. Presumably this happens because (in both ways) of getting this configuration one has to go through a nine (non-compact) dimensional theory that has to be obtained by a non-trivial Scherk-Schwarz type compactification from 10 dimensions as discussed in [9]. It would be interesting to investigate this issue further.

In the above we've used compactification of non-compact directions and T-duality at will. However it is worthwhile pointing out that this cannot always be done, by showing how a potential paradox is avoided. This relates to the fact that total charge of a configuration can be non-zero only if the space is non-compact. First consider nine-branes in type IIB string theory. As is well-known the only possibility is when one has imposed a Z_2 projection to get the type I string from IIB so that the corresponding single orientifold plane charge cancels the charge of 16 nine-branes and their images to give an $SO(32)$ theory (see [10] for a review). This is the case even if the space is non-compact. The point is that (for n nine-branes) there will be a term $n \int_{M_{10}} A_{10}$ without a corresponding kinetic term, since $dA_{10} = F_{11} = 0$ identically in 10 dimensions, so that integration over the 10-form gauge field gives $n = 0$ except in the above mentioned type I case. The puzzle is that one seems to be able to obtain a configuration of n nine-branes starting from the corresponding configuration of lower dimensional branes, in a non-compact space, and T dualizing. The resolution is that T-duality requires that the transverse dualized dimension actually be compact, while the existence of an arbitrary number of p-D-branes (with $p < 9$) requires the existence of at least one transverse dimension that is non-compact. Thus for instance consider the case of n parallel 8-D-branes in type IIA perpendicular to the X^9 axis. The relevant piece of the action is

$$\frac{1}{2} \int_{M_{10}} F_{10} \wedge *F_{10} + \sum_{i=1}^n \epsilon^i \mu_8 \int_{W_9^i} A_9 \quad (0.13)$$

where $\epsilon^i = \pm$ and μ_8 is the magnitude of the eight-brane charge and W_9^i is the world volume of the i 'th 8-brane. The equation of motion for A_9 is $d * F_{10} = \mu_8 \sum \epsilon^i \delta(M_{10} \rightarrow W_9^i)$. Integrating over a disc transverse to the eight-branes we get

$$Q = \int_{S_\infty^0} *F_{10} = *F_{10}(X^9 = +\infty) - *F_{10}(X^9 = -\infty) = \mu_8 \sum \epsilon^i \quad (0.14)$$

If the ninth dimension is non-compact then clearly we can have non-zero (total) 8-brane charge. But in this case we cannot T-dualize in this direction. On the other hand if this dimension is compact the total charge is zero and we can T-dualize to get the IIB situation discussed earlier. Thus there is no conflict!

We end this note by pointing out that the above mentioned problem does not occur for the T-dualities that were used earlier to relate various brane configurations. In each case that we use T-duality there is at least

one dimension transverse to each brane that remains uncompactified, so that the charge is not forced to vanish.

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